The Calculation of Turbulent Wakes

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Abstract

CALCULATED values of velocity, integral properties, and Reynolds shear stress are compared with available measurements for wake flows. The calculated results were obtained by solving boundary-layer equations with an algebraic eddy-viscosity formula consistent with that examined previously for an extensive range of wall boundary layers. The range of flows encompasses symmetric and asymmetric wakes, a wall-jet arrangement where a wake merges into a boundary layer, a wake in a pressure gradient, and curved wakes associated with airfoils at angle of attack. It is concluded that, provided the cross-stream pressure gradients associated with wake curvatures are represented properly by the form of the momentum equation, the algebraic eddy-viscosity approach is adequate for the wakes considered.

Contents

In the development of a method to calculate the flow over multielement airfoils, it is essential that the abilities of the procedure be appraised in terms of their ability to represent identifiable components of the multielement systems. This synoptic, based on Ref. 1, describes calculated results for wake flows and, together with a companion report on wall flows,[2] quantifies the extent to which the flow over essential components can be described for small angles of attack. It remains, of course, to develop and test a calculation method that will permit the calculation of complete systems and to extend it to deal with the local regions of separation and the highly curved wakes associated with high angles of attack. The calculation scheme described in Ref. 3 goes some way to the achievement of this long-term aim.

The recent comparisons of calculated and measured results for two-dimensional wakes[4] allowed the conclusion that the important characteristics of symmetric and asymmetric wakes can be predicted satisfactorily although the agreement between calculation and experiment deteriorates with distance from the trailing edge and the asymptotic growth rates are underestimated. The calculations were based on the solution of boundary-layer equations and the finite difference method of Ref. 5—the two-equation turbulence model of Ref. 6 was used. The experimental data corresponded to the symmetric wakes of Refs. 7-9 and the asymmetric wakes of Refs. 8 and 9.

The present calculations have also been performed for the data of Refs. 7-9 and, together with the results of Ref. 4, allow comparison of the algebraic and transport-equation eddy-viscosity approaches. In addition, the measurements of Refs. 10-13 are considered and, since the last two involve pressure gradients, the range of applicability of the model is extended. In the case of Ref. 13, the wakes involve cross-stream pressure gradients so that the errors associated with their neglect are evaluated and quantitative information is provided on the need for the inclusion of a reduced form of the v-momentum equation.

The boundary-layer equations were solved with a finite difference method based on Keller's box scheme and the eddy-viscosity formulation

$$\nu_t = \nu_w + (\nu_{te} - \nu_w)e^{-B_1} \tag{1}$$

where ν_{te} is the eddy viscosity at the trailing edge, $B_1 = (x - x_{te})/20\delta_{te}$, and ν_w the eddy viscosity for the far wake given by the maximum of ν_{w_L} and ν_{w_u} defined by

$$\nu_{w_L} = 0.064\int_{-\infty}^{y_{\min}} (u_{e_L} - u)\mathrm{d}y,$$

$$\nu_{w_u} = 0.064\int_{y_{\min}}^{\infty} (u_{e_u} - u)\mathrm{d}y \tag{2}$$

Here the subscript L denotes the lower wake and u the upper wake. The location of $y_{\min}$ corresponds to $u = u_{\min}$. It should be noted that in Eq. (2), the far wake is different from that suggested by Townsend[14] so that it can apply to asymmetric wakes.

It is useful to note that the calculation times associated with the algebraic eddy-viscosity models and the present flows were approximately three times less than those associated with the two-equation model and 10 times less than those with a Reynolds stress model. The calculated and measured growth of the maximum defect velocity and the shape factor and mean velocity and shear-stress profiles are presented in 10 figures which correspond to the flow arrangements of Refs. 7-13. In all cases, the calculations were begun with an initial profile taken from the experimental results at the trailing edge.

In common with most results obtained with the present approach, those for the symmetric wakes of Refs. 7-11 are in close agreement with the measurements, but small discrepancies occur in the upstream and downstream regions. In the upstream region, the calculated results suggest more rapid mixing than the measurements even though the model does not represent the effects of the finite trailing-edge thickness or local eddy shedding and high mean-velocity results can be recorded in regions of high turbulence intensity, such as occurs close to the trailing edge. In the downstream region, the discrepancies are small and appear to be due mainly to asymmetry in the measured wake. As would be expected, the variation of the presumed symmetry-line velocity and the variation of integral quantities represented here by the shape factor H are well described by the calculation to the mean-velocity profiles. Mean-velocity information was also reported in Ref. 8 for a wall-jet-type flow and the calculated results are again in close agreement. The results for the asymmetric wake of Ref. 8, with the exception of the near wake ($x = 25$ mm) where the rise of the minimum velocity is again overpredicted and the maximum shear stress is slightly underpredicted, are within the experimental uncertainty: the shear-stress results are particularly encouraging, as can be seen from Fig. 1.

It is evident from the results that the algebraic eddy-viscosity formulation allows values of mean velocity and shear stress to be calculated for symmetric and asymmetric wakes with precision almost within experimental error. Larger discrepancies tend to correspond to the very near wake and are

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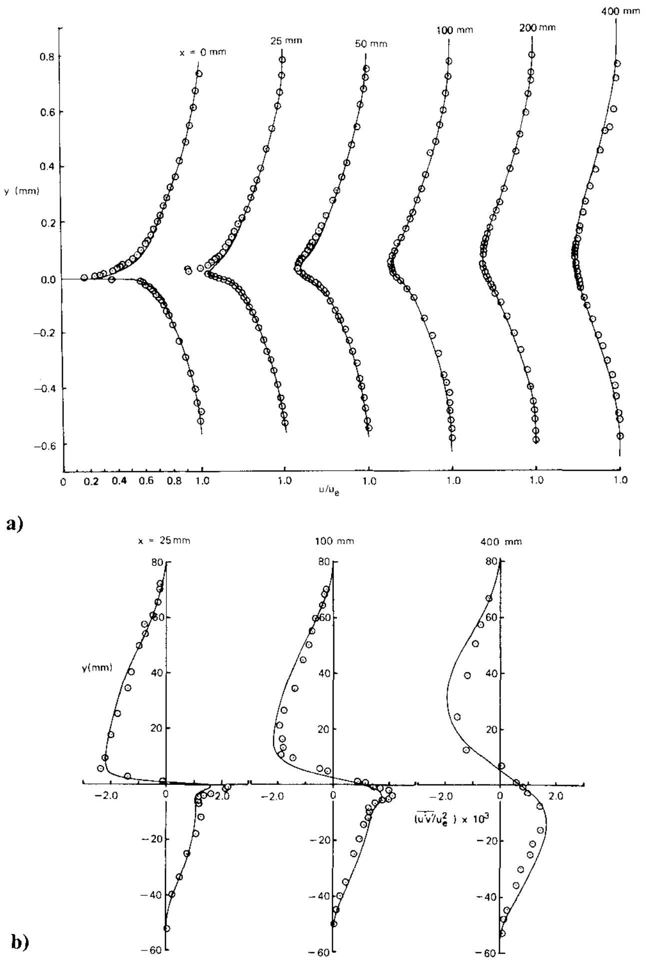

Fig. 1 Asymmetric wake of Ref. 8: a) velocity profiles b) Reynolds shear stress.

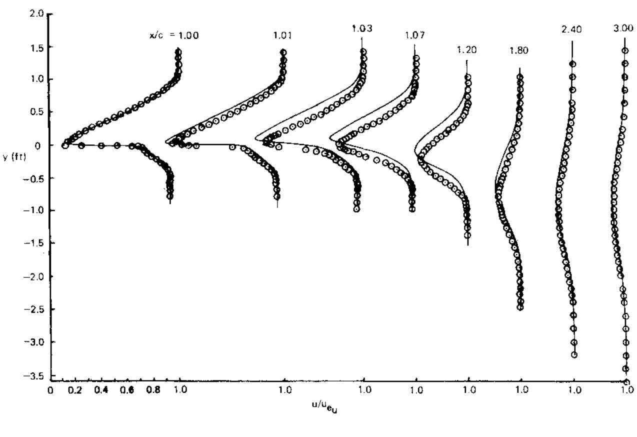

Fig 2 Asymmetric wake of Ref. 14.

probably due to the influence of the geometry of the trailing edge and the local presence of coherent structures. In their conclusions, Patel and Scheurer[4] suggested similar merits for the k-ϵ model but added the caveat that the agreement between prediction and experiment deteriorates with distance from the trailing edge and that asymptotic growth rates are underestimated. Here we are not concerned with wakes beyond x/θ of 400 and are encouraged that the algebraic eddy-viscosity formulation provides results that are as good as those obtained with a two-equation eddy-viscosity model and at a relative calculation cost of less than 0.3.

In aerodynamic applications, pressure gradients are often important and the remaining results are concerned with flows where they exist. The results of Ref. 13 were obtained, in part, with longitudinal adverse pressure gradients and, although three-dimensional effects are likely to increase in importance with distance from the trailing edge, the results again suggest that the present effective-viscosity approach is adequate even in a flow where the freestream velocity decreases from around 135 to 76 ft/s in 1.83 ft.

Wake curvature involves cross-stream pressure gradients and the results of Ref. 13 suggest that these require consideration if calculated results are to be correct. Calculations were performed with the experimental wake trajectory and pressure gradient term $(\partial p/\partial y)$ and the results show that the rise of the minimum velocity and decay of shape factor are well represented by the present approach. The momentum thickness is up to 10% high—this is consistent with the small discrepancies in velocity.

Figure 2 shows that, with the pressure field represented correctly, the agreement between measurements and calculations is marginally poorer than that with the symmetric and asymmetric wakes previously considered. This suggests that the algebraic eddy-viscosity approach is adequate for the calculation of the wakes considered here. It is also evident that a truly predictive scheme should consider the cross-stream momentum equation, perhaps in the form of an interaction between the viscous- and inviscid-flow equations.

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